Jet Theory

Precision Jet Physics Using Soft-Collinear Effective Theory

Yang-Ting Chien

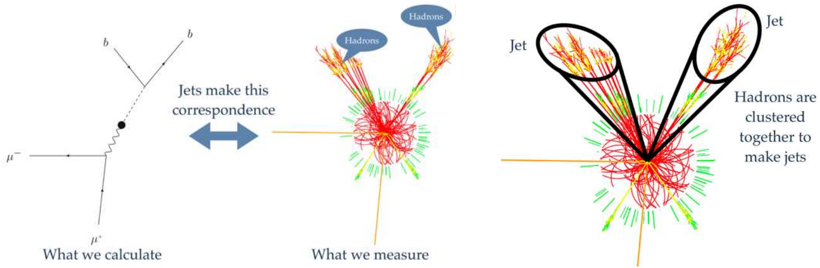
Los Alamos National Laboratory, Theoretical Division, T-2

June 8, 2016
RHIC & AGS Annual Users' Meeting
Brookhaven National Laboratory

Outline

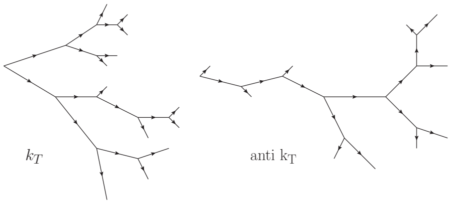
- Jets and jet substructure
  - Resolve QCD radiations with jet observables
  - Power counting soft and collinear radiations
  - The need of resummation
- Soft-collinear effective theory (SCET)
  - Factorization theorem
  - Renormalization group evolution
  - Medium modification by Glauber interactions
- Conclusions

Jets and QCD



- Jets are manifestation of the underlying colored partons
- Jet clustering algorithms merge the pair of particles with the shortest distance until the angular cutoff R
- the distance measure d_{ij} between particles i and j is defined by

$$d_{ij} = \min(p_{ti}^{2\beta}, p_{tj}^{2\beta})\Delta R_{ij}^2/R^2$$



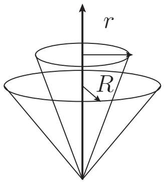
$\beta = 1$: k_T $\beta = 0$, C/A $\beta = -1$, anti-k_T

QCD and effective field theory

Systematically decompose QCD radiations

- Resolve jets at different energy scales
  - To zeroth order, the jet kinematics corresponds to the parton kinematics
  - A jet is not simply a parton but with sequential branching and splitting
  - One needs to measure substructure to study the jet formation mechanism
- The dominant contributions to jet observables come from radiations which are
  - Energetic, *collinear*
  - *Soft*, ubiquitous (not necessarily collinear)
- Power counting by systematically defining *collinearity* and *softness*
  - It is like dimensional analysis which is the first thing a physicist should do

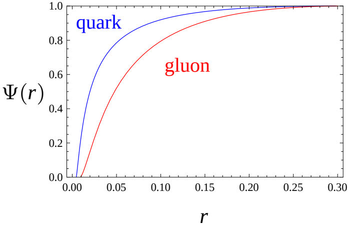
Jet shape, a classic jet substructure observable (Ellis, Kunszt, Soper)

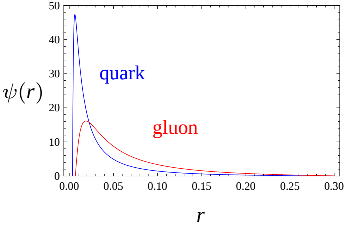


$$\Psi_J(r,R) = \frac{\sum_{r_i<r} E_{T\,i}}{\sum_{r_i<R} E_{T\,i}}$$

$$\langle \Psi \rangle = \frac{1}{N_J} \sum_{J}^{N_J} \Psi_J(r,R)$$

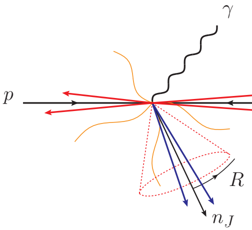
$$\psi(r,R) = \frac{d\langle \Psi \rangle}{dr}$$



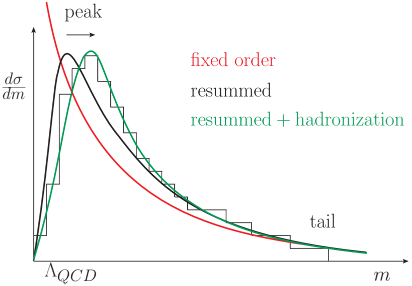


- Jet shapes probe the averaged energy distribution inside a jet
- The infrared structure of QCD induces Sudakov logarithms
- Fixed order calculation breaks down at small r
- Large logarithms of the form $\alpha_s^n \log^m r/R \ (m \leq 2n)$ need to be resummed

Jet mass, the simplest and most important substructure observable



$$m^2 = \Big(\sum_{i\in J} p_i\Big)^2$$
$$= \big(p_c + p_s\big)^2$$
$$\approx p_c^2 + 2p_c \cdot p_s$$
$$\approx p_c^2 + 2E_J n_J \cdot p_s$$

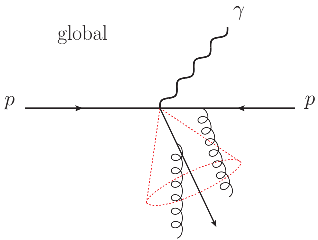


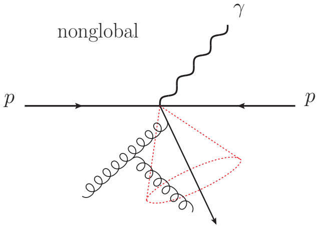
- Jet mass is a soft radiation sensitive jet substructure observable
- The infrared structure of QCD induces Sudakov logarithms
- Fixed order calculation breaks down at small jet mass m
- Large logarithms of the form

$$\frac{1}{m}\alpha_s^i\Big(\log^j \frac{m}{E_J} \text{ or } \log^j R\Big), \quad j \le 2i-1$$

need to be resummed
- Hadronization affects the position of the peak at small m
- Resummation of $\log R$ is crucial especially for jets with small radii

Resummation precision





$$\frac{1}{r}\alpha_s^i\left(\log^j\frac{r}{R}\right) \text{ or } \frac{1}{m}\alpha_s^i\left(\log^j\frac{m}{E_J},\ \log^j R\right), \quad j \leq 2i-1$$

- *All-order* resummation: $i = 1, \ldots \infty$
- Infrared structure of QCD allows the all-order resummation of logarithmically enhanced terms without calculating diagrams at all orders
  - leading-logarithmic (LL) accuracy: $j = 2i - 1$
  - next-to-leading-logarithmic (NLL) accuracy: $j = 2i - 1, 2i - 2$
  - ...
- Nonglobal logs and clustering logs appear at NNLL
  - Resummation is still an open question
  - Groomed jet observable is a way out

Resummation and effective field theory

THE BASIC IDEA

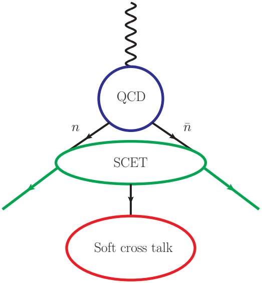
- Logarithms of *scale ratios* appear in perturbative calculations
  - Logarithms become large when scales become hierarchical

$$\log\frac{r}{R} = \log\frac{\text{scale 1}}{\text{scale 2}}, \qquad \log\frac{m}{E_J}, \qquad \log R = \log\frac{\text{scale 3}}{\text{scale 4}}$$

- In effective field theories, logarithms are resummed using renormalization group evolution between characteristic scales
  - To resum *all* the logarithms we need to identify *all* the relevant scales in EFT

Soft-Collinear Effective Theory (SCET)

- Effective field theory techniques are most useful when there is clear scale separation
- SCET separates physical degrees of freedom in QCD by a systematic expansion in power counting
  - Match SCET with QCD at the hard scale by integrating out the hard modes
  - Integrating out the off-shell modes gives collinear Wilson lines which describe the collinear radiation
  - The soft sector is described by soft Wilson lines along the jet directions

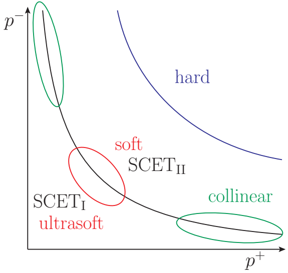


- At leading power, soft-collinear decoupling holds in the Lagrangian and it leads to the factorization of cross sections

Power counting in SCET

- The scaling of modes in lightcone coordinates $(\bar{n} \cdot p, n \cdot p, p_\perp)$ where $n = (1, 0, 0, 1)$ and $\bar{n} = (1, 0, 0, -1)$:

$$p_h : E_J(1,1,1),\ p_c : E_J(1,\lambda^2,\lambda) \text{ and } p_s : E_s(1,R^2,R)$$



  - E_J is the hard scale which is the energy of the jet
  - λ is the **power counting** parameter ($\lambda \approx m/E_J$)
  - $E_J\lambda$ is the jet scale which is significantly lower than E_J
  - The relevant soft scales depend on observables
- QCD $= \mathcal{O}(\lambda^0) + \mathcal{O}(\lambda^1) + \cdots$ in SCET
  - Leading-power contribution in SCET is a very good approximation

Power counting jet observables

Determine how collinear and soft radiations contribute

- Jet shapes have dominant contributions from the collinear sector

$$\Psi(r) = \frac{E_c^{<r} + E_s^{<r}}{E_c^{<R} + E_s^{<R}} = \frac{E_c^{<r}}{E_c^{<R}} + \mathcal{O}(\lambda)$$

  - Soft contributions are power suppressed
- Jet mass is sensitive to c-soft modes: ultrasoft modes constrained inside jets

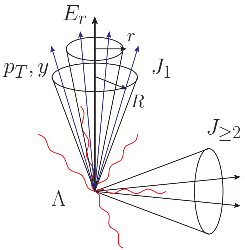
$$m^2 = (p_c + p_s)^2 \approx p_c^2 + 2E_J n_J \cdot p_s \approx E_J^2 \lambda^2, \qquad E_s = E_J \frac{\lambda^2}{R^2} = \frac{m^2}{E_J R^2}$$

$$p_c : E_J(1, \lambda^2, \lambda) \text{ and } p_s : E_s(1, R^2, R)$$

- For high p_T and narrow jets, power corrections are small and the leading power contribution is a very good approximation of the full QCD result

Factorization theorem for jet shapes (Chien et al)

- Without loss of generality, we demonstrate the calculation in e^+e^- collisions since the initial state radiation in proton collisions contributes as power corrections



- The factorization theorem for the differential cross section of the production of N jets with $p_{T\,i}$, y_i, the energy E_r inside the cone of size r in one jet, and an energy cutoff Λ outside all the jets is the following,

$$\frac{d\sigma}{dp_{T\,i}dy_i dE_r} = H(p_{T\,i}, y_i, \mu) J_1^{\omega_1}(E_r, \mu) J_2^{\omega_2}(\mu) \ldots S_{1,2,\ldots}(\Lambda, \mu)$$

- For the differential jet rate (without measuring E_r)

$$\frac{d\sigma}{dp_{T\,i}dy_i} = H(p_{T\,i}, y_i, \mu) J_1^{\omega_1}(\mu) J_2^{\omega_2}(\mu) \ldots S_{1,2,\ldots}(\Lambda, \mu)$$

- $H(p_{T\,i}, y_i, \mu)$ describes the hard scattering process at high energy
- $J_1^{\omega}(E_r, \mu)$ describes the probability of having the amount of energy E_r inside the cone of size r
- $S_{1,2,\ldots}(\Lambda, \mu)$ describes how soft radiation is constrained in measurements
- The factorization theorem has a product form instead of a convolution

Factorization theorem for jet shapes (continued)

The averaged energy inside the cone of size r in jet 1 is the following,

$$\langle E_r \rangle_\omega = \frac{1}{\frac{d\sigma}{dp_{T\,i}dy_i}} \int dE_r E_r \frac{d\sigma}{dp_{T\,i}dy_i dE_r} = \frac{H(p_{T\,i}, y_i, \mu) J^{\omega_1}_{E,r_1}(\mu) J^{\omega_2}_2(\mu) \ldots S_{1,2,\ldots}(\Lambda, \mu)}{H(p_{T\,i}, y_i, \mu) J^{\omega_1}_1(\mu) J^{\omega_2}_2(\mu) \ldots S_{1,2,\ldots}(\Lambda, \mu)} = \frac{J^{\omega_1}_{E,r_1}(\mu)}{J^{\omega_1}_1(\mu)}$$

- $J^{\omega}_{E,r}(\mu) = \int dE_r E_r J^{\omega}(E_r, \mu)$ is referred to as the jet energy function
- Nice cancelation between the hard, "unmeasured" jet and soft functions
- The integral jet shape, averaged over all jets, is the following

$$\langle \Psi \rangle = \frac{1}{\sigma_{\text{total}}} \sum_{i=q,g} \int_{PS} dp_T dy \frac{d\sigma}{dp_T dy} \Psi^i_\omega \ , \ \text{where } \Psi_\omega = \frac{J_{E,r}(\mu)/J(\mu)}{J_{E,R}(\mu)/J(\mu)} = \frac{J_{E,r}(\mu)}{J_{E,R}(\mu)}$$

Factorization theorem for jet mass (Chien et al)

- The cross section differential in photon p_T, y, and jet mass m can be first factorized as a convolution with parton distribution functions

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}p_T\mathrm{d}y\mathrm{d}m^2} = \frac{2}{p_T}\sum_{ab}\int dvdw \;\; x_1 f_a(x_1,\mu)\; x_2 f_b(x_2,\mu)\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}w\mathrm{d}v\mathrm{d}m^2} \;,$$

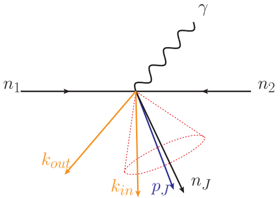
where

$$x_1 = \frac{1}{w}\frac{p_T}{E_{CM}v}e^y, \qquad x_2 = \frac{p_T}{E_{CM}(1-v)}e^{-y}$$

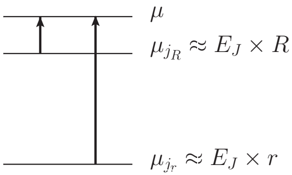
- The partonic cross section can be further factorized in SCET as a convolution of the hard, jet and soft function

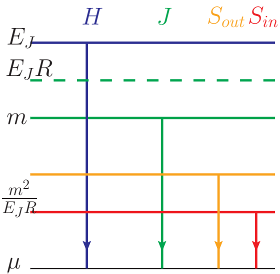
$$\begin{aligned}\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}w\mathrm{d}v\mathrm{d}m^2} \quad &= \quad w\,\hat{\sigma}(v)\;H(p_T,v,\mu)\int \mathrm{d}k_{in}\mathrm{d}k_{out}\mathrm{d}p^2 J(p^2,\mu)S(k_{in},k_{out},\mu) \\ &\quad \times\delta(m^2-p^2-2E_Jk_{in})\delta(m_X^2-m^2-2E_Jk_{out})\end{aligned}$$

where $m_X^2 = (p_J + k_{in} + k_{out})^2$ is the partonic mass of the event



Scale hierarchy and renormalization group evolution



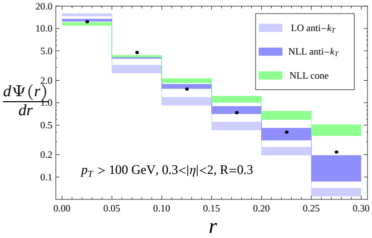


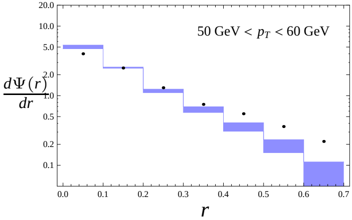
- Each factorized piece $\mathcal{O}$ captures physics at certain characteristic scale $\mu_{\mathcal{O}}$
- The renormalization group evolution between characteristic scales resums the logs of the scale ratios

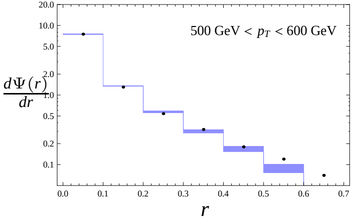
$$\mu \frac{d\mathcal{O}}{d\mu} = \gamma_{\mathcal{O}} \mathcal{O}$$

  - The anomalous dimension $\gamma_{\mathcal{O}}$ can be calculated order-by-order in perturbation theory
  - $\log r/R = \log \mu_{j_r}/\mu_{j_R}$

Results for jet shapes



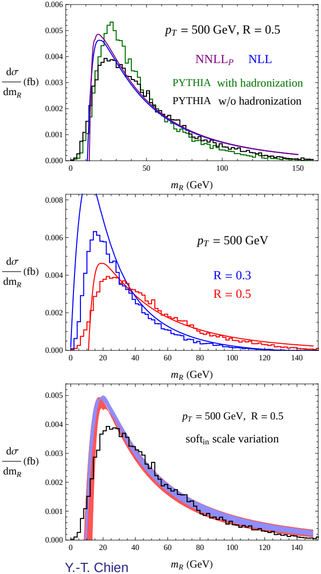




- Compare with CMS pp data at 2.76 and 7 TeV
- The difference for jets reconstructed using different jet algorithms is of $\mathcal{O}(r/R)$
- Bands are theory uncertainties estimated by varying μ_{j_r} and μ_{j_R}
- In the region $r \approx R$ we may need higher fixed order calculations and include power corrections

- NLL, anti-k_T, $R = 0.7$
- For low p_T jets, power corrections have significant contributions

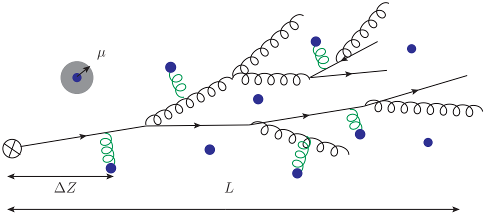
Results for jet mass



- The most precise analytic calculation of jet mass distributions to date
- Agree nicely with PYTHIA partonic calculation within theoretical uncertainty
  - Comparison with data will be performed
- Hadronization effect plays a role as shown in PYTHIA simulations
  - Analytic study of nonperturbative soft matrix element will be included
- Jet radius dependence correctly captured

Multiple scattering in a medium

- Coherent multiple scattering and induced bremsstrahlung are the qualitatively new ingredients in the medium parton shower
- Interplay between several characteristic scales:
  - Debye screening scale μ
  - Parton mean free path λ
  - Radiation formation time τ

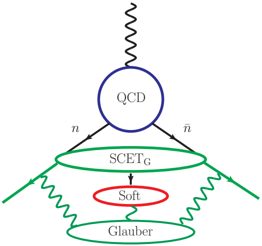


- From thermal field theory and lattice QCD calculations, an ensemble of quasi particles with debye screened potential and thermal masses is a reasonable parameterization of the medium properties

$$\frac{1}{\sigma_{el}}\frac{d\sigma_{el}}{d^2q_\perp} = \frac{\mu^2}{\pi(q_\perp^2+\mu^2)^2}$$

SCET with Glauber gluons ($\mathrm{SCET_G}$)

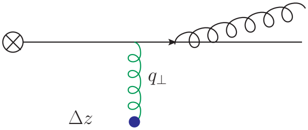
- Glauber gluon is the relevant mode for medium interactions
- $\mathrm{SCET_G}$ was constructed from SCET bottom up (Idilbi et al, Vitev et al)
    - Glauber momentum scales as $p_G : Q(\lambda^2, \lambda^2, \lambda)$
    - Glauber gluons are off-shell modes providing momentum transfer transverse to the jet direction
    - Glauber gluons are treated as background fields generated from the color charges in the QGP
    - Glauber gluons interact with both the collinear and the soft modes
- Given a medium model, we can use $\mathrm{SCET_G}$ to consistently couple the medium to jets



Medium-induced splitting

- The hierarchy between τ and λ determines the degree of coherence between multiple scatterings

$$\tau = \frac{x\,\omega}{(q_\perp - k_\perp)^2} \quad \text{v.s.} \quad \lambda$$



- Medium-induced splitting functions were calculated using $\mathrm{SCET_G}$ (Ovanesyan et al)

$$\frac{dN^{med}_{q\to qg}}{dxd^2k_\perp} = \frac{C_F\alpha_s}{\pi^2}\frac{1}{x}\int_0^L \frac{d\Delta z}{\lambda}\int d^2q_\perp \frac{1}{\sigma_{el}}\frac{d\sigma_{el}}{d^2q_\perp}\frac{2k_\perp\cdot q_\perp}{k_\perp^2(q_\perp-k_\perp)^2}\left[1-\cos\left(\frac{(q_\perp-k_\perp)^2\Delta z}{x\omega}\right)\right]$$

- $\frac{dN^{med}}{dxd^2k_\perp} \to$ finite as $k_\perp \to 0$: the Landau-Pomeranchuk-Migdal effect

Jet shapes in heavy ion collisions (continued)

- Jet shapes get modified through the modification of jet energy functions

$$\Psi(r) = \frac{J^{vac}_{E,r} + J^{med}_{E,r}}{J^{vac}_{E,R} + J^{med}_{E,R}} = \frac{\Psi^{vac}(r) J^{vac}_{E,R} + J^{med}_{E,r}}{J^{vac}_{E,R} + J^{med}_{E,R}}$$

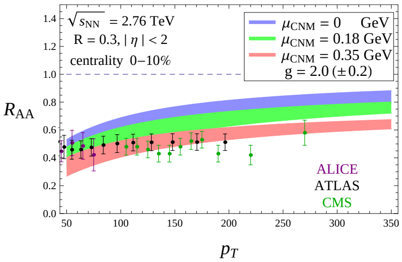
  - Large logarithms in $\Psi^{vac}(r) = J^{vac}_{E,r}/J^{vac}_{E,R}$ have been resummed
  - There are no large logarithms in $J^{med}_{E,r}$ due to the LPM effect
  - The RG evolution of medium-modified jet energy functions is unchanged
- However, with the use of small R's in heavy ion collisions, there is significant jet energy loss which leads to the suppression of jet production cross sections
- Jet-by-jet shapes are averaged with the jet cross sections

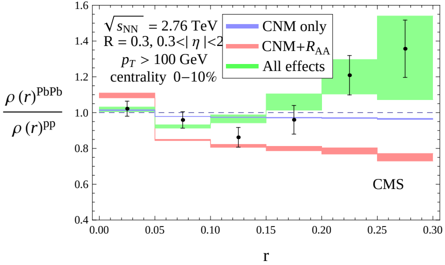
$$\frac{1}{\langle N_{\rm bin}\rangle}\frac{d\sigma^k_{\rm CNM}}{d\eta dp_T} = \sum_{ijX}\int dx_1 dx_2 f_i^A(x_1,\mu_{\rm CNM}) f_j^A(x_2,\mu_{\rm CNM}) \frac{d\sigma_{ij\to kX}}{dx_1 dx_2 d\eta dp_T}$$

$$\frac{1}{\langle N_{\rm bin}\rangle}\frac{d\sigma^i_{med}}{d\eta dp_T}\bigg|_{p_T} = \frac{1}{\langle N_{\rm bin}\rangle}\frac{d\sigma^i_{\rm CNM}}{d\eta dp_T}\bigg|_{\frac{p_T}{1-\epsilon_i}} \frac{1}{1-\epsilon_i}$$

  - With cold nuclear matter effects in nuclear parton distributions

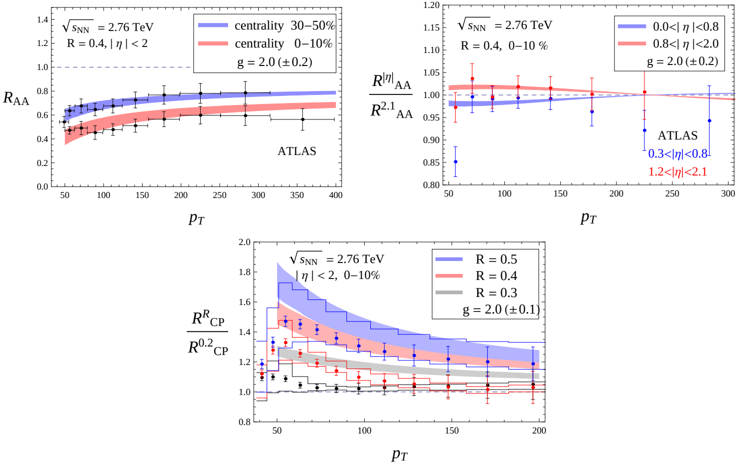
Results





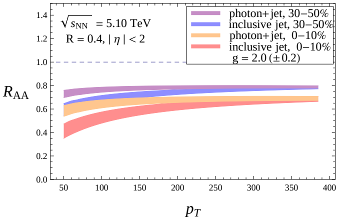
- The plots are the ratios between the jet cross sections and differential jet shapes in lead-lead and proton collisions
- Jet shapes are insensitive to cold nuclear matter effects
- Gluon jets are more suppressed which increases the quark jet fraction
- Jet-by-jet the shape is broadened

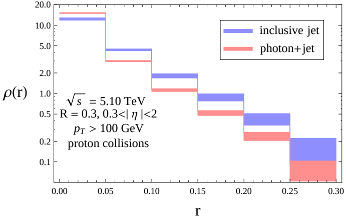
Results

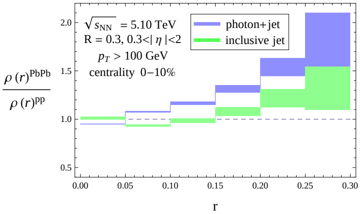


- The plots shows the dependence of jet cross section suppressions on centrality, jet rapidity and jet radius

Results





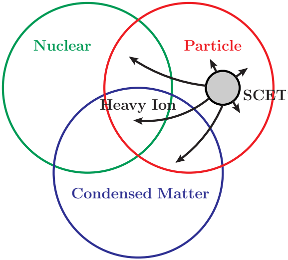


- Predictions for jet shapes and cross sections at 5 TeV for inclusive and photon-tagged jets

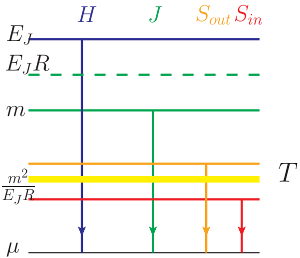
Conclusions

- Jet substructure in proton and heavy ion collisions can be calculated within the same framework
  - Promising agreement with data and simulations and phenomenological applications
  - Stay tuned before Hard Probes 2016 for pA and AA jet fragmentation function and jet mass distribution
- The modification of jet substructure is a combination of cross section suppression and jet-by-jet broadening
- Power counting jet observables is useful and insightful. Give it a try!

Outlooks



- The physics of heavy ion collisions is a multi-disciplinary subject
- The study of jet quenching is a unique opportunity to probe non-perturbative QCD physics with perturbative objects
- Effective field theory techniques can make important contributions



Outlooks

We welcome discussions and requests for calculations

Thank you